



Progression through calculations – Mental Calculations



These are a selection of mental calculation strategies which are taught throughout school and are ongoing. By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Mental recall of number bonds

$$6 + 4 = 10$$

$$25 + 75 = 100$$

$$\square + 3 = 10$$

$$19 + \square = 20$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55 \qquad 19 + 36 = 55$$

$$55 - 19 = 36 \qquad 55 - 36 = 19$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 3 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 8 times table
 10 times table

Year 4 Derive and recall all multiplication facts up to 12×12

Years 5 & 6 Derive and recall quickly all multiplication facts up to 10×10 .

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

$30 \times 7 = 210$, $300 \times 7 = 2,100$, $3,000 \times 7 = 21,000$, $0.3 \times 7 = 2.1$ etc

Partitioning

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

Multiplication

Subtraction

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 3 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 8 times table
 10 times table

Year 4 Derive and recall division facts for all tables up to 12×12

Year 5 & 6 Derive and recall quickly division facts for all tables up to 12×12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know? e.g. $21 \div 7 = 3$, $7 \times 3 = 21$ etc.

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

Division



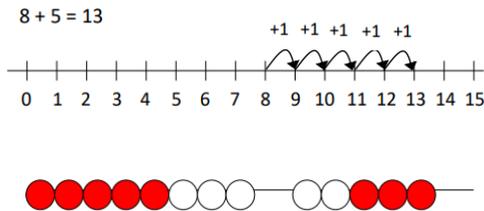
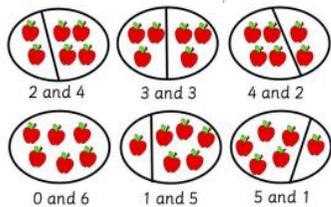
Progression through calculations – Year R and Year 1



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

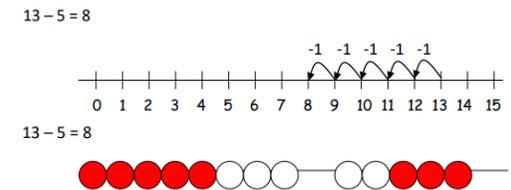
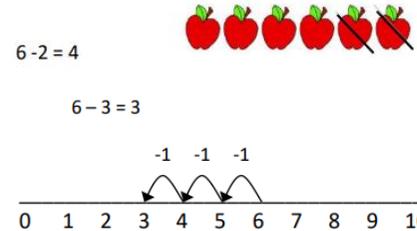
Addition

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They do this by using Numicon resources and other practical equipment. They also use number lines to support calculation and teachers demonstrate the use of these. Bead strings or bead bars can also be used to illustrate addition.

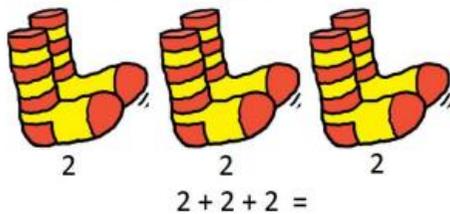


Subtraction

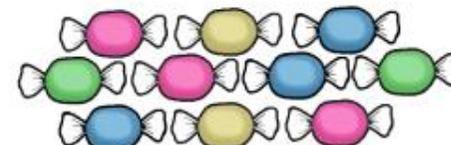
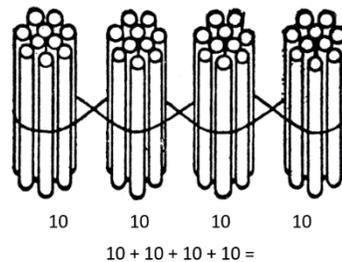
Children use lots of practical apparatus to develop a mental picture of the number system in their heads to use for calculation. They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away. Children will begin to count back from a given number. They use number lines and practical resources to support calculation. Teachers demonstrate the use of the number line. Bead strings or bead bars can be used to illustrate subtraction.



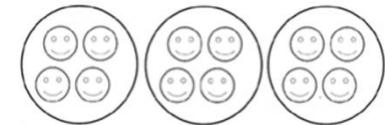
How many socks?



How many sticks?



I have 10 sweets. They are shared with 2 people. How many sweets will they have each?



How many stickers are in each group?

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem-solving activities involving equal sets or groups.

Children will understand equal groups and sharing items equally out in play and problem solving. They will count in 2s and 10s and later in 5s.

Multiplication

Division



Progression through calculations – Year 2



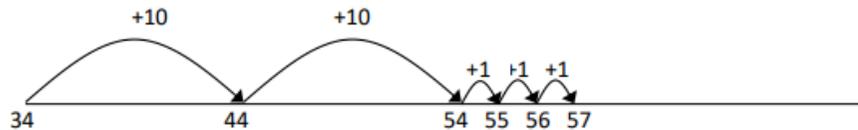
By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Children will then begin to use 100 squares to carry out addition, learning to move down the columns when adding on tens and to the right for adding on ones. Children will then move onto using pre-printed number lines (up to 50) progressing to using 'empty number lines' starting with the larger number and counting on.

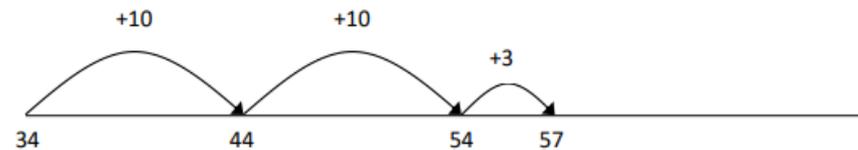
- First counting on in tens and ones.

$34 + 23 = 57$



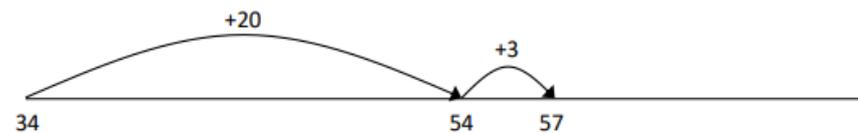
- Then helping children to become more efficient by adding the ones in one jump (by using the known fact $4 + 3 = 7$).

$34 + 23 = 57$



- Followed by adding the tens in one jump and the ones in one jump.

$34 + 23 = 57$



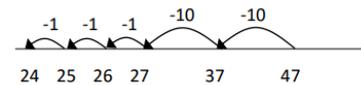
Subtraction

Children will use 100 squares and pre-prepared number lines to 30 and to 50 to help support their subtraction calculations. They will then begin to use empty number lines to support calculations.

Counting back

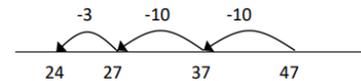
- First counting back in tens and ones.

$47 - 23 = 24$



- Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact $7 - 3 = 4$).

$47 - 23 = 24$



- Subtracting the tens in one jump and the ones in one jump.

$47 - 23 = 24$

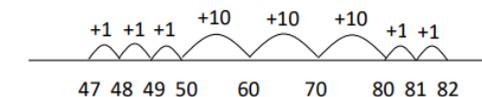


Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

$82 - 47$





Progression through calculations – Year 2



Multiplication

Children will develop their understanding of multiplication through lots of practical hands on examples and will then use jottings to support calculation:

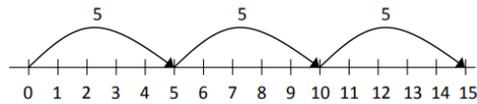
- **Repeated addition**

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

This can be shown by using practical apparatus such as multilink or counters etc.

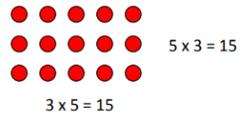
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$

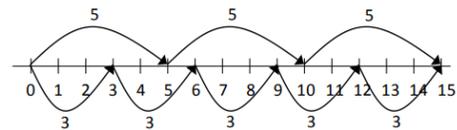


- **Commutativity**

Children should know that 3×5 has the same answer as 5×3 . This can be shown using an array;

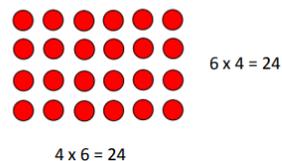


and then on a number line.



- **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



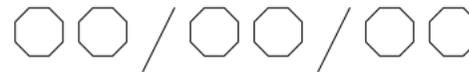
Division

Children will develop their understanding of division and use jottings to support calculation:

- **Grouping or repeated subtraction**

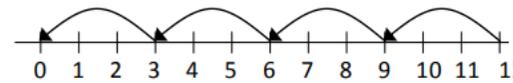
This is taught practically through the use of a variety of apparatus.

There are 6 sweets, how many people can have 2 sweets each?



- **Repeated subtraction using practical apparatus, such as multilink, and then a number line**

$$12 \div 3 = 4$$



The children will be taught to interpret division calculations such as $10 \div 5$ as 'how many groups of 5 make 10?'

- **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

These will be solved through using arrays but ideally children will eventually be able to solve them using their multiplication facts. They will begin to recognise the number families involved.



Progression through calculations – Year 3



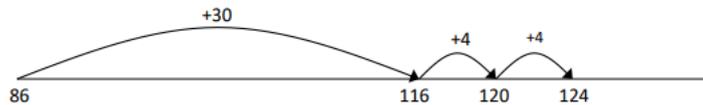
By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate. Children will be encouraged to use various visual aids to secure their knowledge of place value.

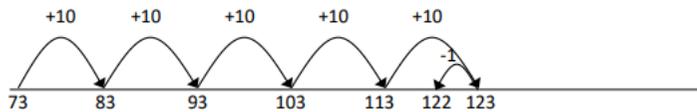
- Count on from the largest number irrespective of the order of the calculation.

$38 + 86 = 124$



- Compensation

$49 + 73 = 122$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Children will then move on from number lines to a more formal written method making sure that the children always write the largest number at the top and then add the least significant digits (ones) first. From this, children will begin to exchange across the ten or hundred below the calculation numbers but above the totals line.

$126 + 53 = 179$

$e = 180$

$175 + 147 = 322$

$e = 300$

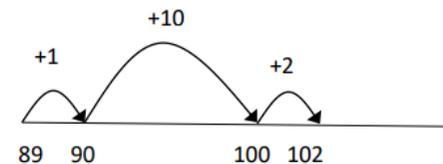
	H	T	O
	1	2	6
+		5	3
	1	7	9

	H	T	O	
	1	7	5	
+		1	4	7
	1	1		
	3	2	2	

Subtraction

Children will continue to use empty number lines with increasingly large numbers. They will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies. Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc. counting on using a number line should be used.

$102 - 89 = 13$



Children will then move on from number lines to a more formal written method making sure that the children always write the largest number at the top and then add the least significant digits (ones) first. Children will always estimate before attempting to calculate.

$157 - 26 = 131$

$e = 130$

	H	T	O
	1	5	7
-		2	6
	1	3	1

The children will be taught to 'exchange' 1 ten for 10 ones, or 1 hundred for 10 tens etc.

$182 - 58 = 124$

$e = 120$

	H	T	O
		7	12
-		1	8
		5	8
	1	2	4



Progression through calculations – Year 3

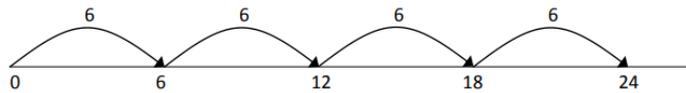


Multiplication

Children will continue to use:

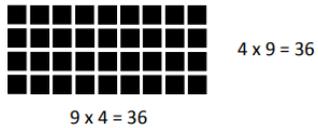
- **Repeated addition**

4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4



- **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Children will also develop an understanding of:

- **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



- **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20 \quad 3 \times \triangle = 18 \quad \square \times \bigcirc = 32$$

- **Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

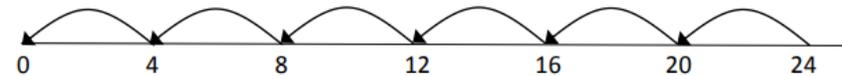
Division

Teachers will ensure that the emphasis in Year 3 is on grouping rather than sharing. Children will continue to use:

- **Repeated subtraction using a number line**

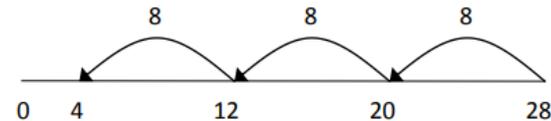
Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$



Children should also move onto calculations involving remainders.

$$28 \div 8 = 3 \text{ r } 4$$



- **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$26 \div 2 = \square \quad 24 \div \triangle = 12 \quad \square \div 10 = 8$$



Progression through calculations – Year 4



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Children will continue to use column addition to calculate. Children will exchange across the ten, hundred or thousand below the calculation numbers but above the totals line. Children will always estimate before attempting to calculate.

$$783 + 142 = 925$$

$$e = 900$$

H	T	O
7	8	3
+	1	4
	<u>1</u>	
9	2	5

$$5,347 + 3,982 = 9,329$$

$$e = 9,000$$

Th	H	T	O
5	3	4	7
+	3	9	8
	<u>1</u>	<u>1</u>	
9	3	2	9

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- know that the decimal points should line up under each other, particularly when adding mixed amounts, e.g. £3.59 + 78p.

Subtraction

Children will continue to use column subtraction to calculate. The children will be taught to 'exchange' 1 ten for 10 ones, or 1 hundred for 10 tens, or 1 thousand for 10 hundreds.

$$754 - 86 = 668$$

$$e = 660$$

H	T	O
	14	
6	4	14
-	7	5
	8	6
6	6	8

$$6,235 - 2,478 = 3,757$$

$$e = 3,700$$

Th	H	T	O
	11	12	
5	1	2	15
-	6	2	3
	2	4	7
3	7	5	7

Using similar methods, Children will:

- be able to subtract numbers with different numbers of digits;
- using this method, begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
- know that decimal points should line up under each other.



Progression through calculations – Year 4



Multiplication

Children will continue to use arrays where appropriate leading into the grid method of multiplication.

x	10	4
6	60	24

$$(6 \times 10) + (6 \times 4)$$

$$\begin{array}{r} 60 \\ + 24 \\ \hline 84 \end{array}$$

Grid method

HTO x O

(Short multiplication – multiplication by a single digit)

143×6

Children will approximate first:

143 x 6 is approximately 150 x 6 = 900

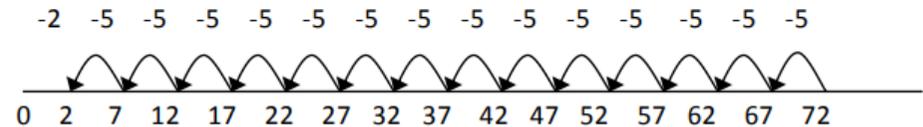
x	100	40	3
6	600	240	18

	100s	10s	1s
	6	0	0
	2	4	0
+		1	8
	<u>8</u>	<u>5</u>	<u>8</u>

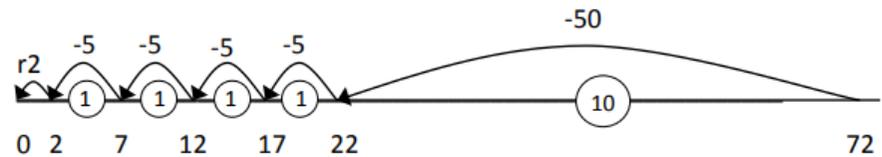
Division

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

$72 \div 5 = 14 \text{ r } 2$



Moving onto:



Leading to:

Short division T O ÷ O

$72 \div 3 = 24$

$$\begin{array}{r} 3 \overline{) 72} \\ - 30 \\ \hline 42 \\ - 30 \\ \hline 12 \\ - 12 \\ \hline 0 \end{array}$$

10 x 3
10 x 3
4 x 3

$457 \div 6 = 76$

$$\begin{array}{r} 6 \overline{) 456} \\ - 300 \\ \hline 156 \\ - 120 \\ \hline 36 \\ - 36 \\ \hline 0 \end{array}$$

50 x 6
20 x 6
6 x 6

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \hline \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \hline \end{array}$$



Progression through calculations – Year 5



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Children will continue to use column addition to calculate. Children will always estimate before attempting to calculate.

$$27,828 + 13,457 = 41,285 \quad e = 41,000$$

	TTh	Th	H	T	O
27,828	2	7	8	2	8
+ 13,457	1	3	4	5	7
	<u>1</u>	<u>1</u>		<u>1</u>	
	4	1	2	8	5

$$794,815 + 273,509 = 1,068,324 \quad e = 1,100,000$$

	M	HTh	TTh	Th	H	T	O
794,815		7	9	4	8	1	5
+ 273,509		2	7	3	5	0	9
		<u>1</u>		<u>1</u>		<u>1</u>	
	1	0	6	8	3	2	4

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding mixed amounts, e.g. 3.2 m + 280 cm

Subtraction

Children will continue to use column subtraction to calculate. Children will always estimate before attempting to calculate.

$$52,752 - 35,384 = 17,368 \quad e = 17,000$$

	TTh	T	H	T	O
52,752	5	2	7	5	2
- 35,384	3	5	3	8	4
		2	7	5	2
		12	6	14	12
	4	12	6	5	2
	<u>1</u>	<u>7</u>	<u>3</u>	<u>6</u>	<u>8</u>

$$352,752 - 135,384 = 217,368 \quad e = 215,000$$

	HTh	TTh	T	H	T	O
352,752	3	5	2	7	5	2
- 135,384	1	3	5	3	8	4
		5	2	7	5	2
		12	12	14	12	
	2	17	3	6	8	
	<u>2</u>	<u>1</u>	<u>7</u>	<u>3</u>	<u>6</u>	<u>8</u>

Using similar methods, children will:

- be able to subtract numbers with different numbers of digits;
- begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other.



Progression through calculations – Year 6



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Children should not be made to go onto the next stage if they are not ready or they are not confident.

Addition

Children will continue to use column addition to calculate. Children should extend the exchanging method to numbers with any number of digits.

$$794,815 + 273,509 = 1,068,324 \quad e = 1,100,000$$

	M	HTh	TTh	Th	H	T	O
		7	9	4	8	1	5
+		2	7	3	5	0	9
		<u>1</u>		<u>1</u>		<u>1</u>	
	<u>1</u>	<u>0</u>	<u>6</u>	<u>8</u>	<u>3</u>	<u>2</u>	<u>4</u>

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to four digits and either one or two decimal places;
- know that decimal points should line up under each other, particularly when adding mixed amounts, e.g. $405.2 + 26.85 + 0.786$.

$$405.2 + 26.85 + 0.786 = 432.836 \quad e = 433$$

	100s	10s	1s	¹ / ₁₀	¹ / ₁₀₀	¹ / ₁₀₀₀
	4	0	5	. 2	0	0
+		2	6	. 8	5	0
			0	. 7	8	6
		<u>1</u>	<u>1</u>	<u>1</u>		
	<u>4</u>	<u>3</u>	<u>2</u>	<u>. 8</u>	<u>3</u>	<u>6</u>

Subtraction

Children will continue to use column subtraction to calculate. Children will always estimate before attempting to calculate.

$$52,752 - 35,384 = 17,368 \quad e = 17,000$$

	TTh	T	H	T	O
				14	
	4	12	6	4	12
	5	2	7	8	4
-	3	5	3	8	4
	<u>1</u>	<u>7</u>	<u>3</u>	<u>6</u>	<u>8</u>

$$352,752 - 135,384 = 217,368 \quad e = 215,000$$

	HTh	TTh	T	H	T	O
					14	
		4	12	6	4	12
	3	5	2	7	8	4
-	1	3	5	3	8	4
	<u>2</u>	<u>1</u>	<u>7</u>	<u>3</u>	<u>6</u>	<u>8</u>

Using similar methods, children will:

- be able to subtract numbers with different numbers of digits;
- be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- know that decimal points should line up under each other.



Progression through calculations – Year 6



Multiplication

Children will use the compact method for both short (multiplication by a single digit) and long (multiplication by multiple digits) multiplication.

HTO x O and ThHTO x O

$$4,346 \times 8$$

Children will approximate first
 $4,346 \times 8$ is approximately $4,000 \times 8 = 32,000$

	10,000s	1000s	100s	10s	1s
		4	3	4	6
x					8
		2	3	4	
		3	4	7	6
		3	4	7	6
		8	8	8	8

HTO X TO and ThHTO x O

$$456 \times 23$$

Children will approximate first
 456×23 is approximately $500 \times 20 = 10,000$

	10,000s	1000s	100s	10s	1s
			4	5	6
x				2	3
			1	1	
		1	3	6	8
+		9	1	2	0
		1	1		
		1	0	4	8
		1	0	4	8

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

Division

Children will continue to use written methods to solve short division up to ThHTO ÷ O. Children will extend their learning to long division.

Short division Th H T O ÷ O

$$4,738 \div 5 = 947 \text{ r } 3$$

$$\begin{array}{r}
 0 \quad 9 \quad 4 \quad 7 \quad \text{r } 3 \\
 5 \overline{) 4 \quad 7 \quad 3 \quad 8} \\
 \underline{4 \quad 0} \\
 7 \\
 \underline{7 \quad 0} \\
 3 \\
 \underline{3 \quad 0} \\
 8 \\
 \underline{5} \\
 3
 \end{array}$$

Extend to decimals with up to two decimal places.

$$289.2 \div 8 = 36.15$$

$$\begin{array}{r}
 0 \quad 3 \quad 6 \quad . \quad 1 \quad 5 \\
 8 \overline{) 2 \quad 8 \quad 9 \quad . \quad 2 \quad 0} \\
 \underline{2 \quad 0} \\
 8 \\
 \underline{8 \quad 0} \\
 9 \\
 \underline{8 \quad 0} \\
 1 \\
 \underline{0 \quad 8} \\
 2 \\
 \underline{1 \quad 6} \\
 4 \\
 \underline{4 \quad 0} \\
 0
 \end{array}$$

Long division TTh Th H T O ÷ T O

$$23,576 \div 17 = 1,386 \text{ r } 14$$

$$\begin{array}{r}
 0 \quad 1 \quad 3 \quad 8 \quad 6 \\
 17 \overline{) 2 \quad 3 \quad 5 \quad 7 \quad 6} \\
 \underline{1 \quad 7} \\
 6 \quad 5 \\
 \underline{5 \quad 1} \\
 1 \quad 4 \quad 7 \\
 \underline{1 \quad 3 \quad 6} \\
 1 \quad 1 \quad 6 \\
 \underline{1 \quad 0 \quad 2} \\
 1 \quad 4
 \end{array}$$

Children to write out 17 times table

- 17
- 34
- 51
- 68
- 85
- 102
- 119
- 136

$$23,576 \div 17 = 1,386 \frac{14}{17}$$

Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in its lowest terms.